The Effect of a Small Mixing Angle in the Atmospheric Neutrinos *

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Abstract

The effect of matter enhanced neutrino oscillations on atmosheric neutrinos is investigated systematically in the framework of one mass dominant model of three neutrinos. The resonance conditions of neutrino crossing the earth are determined by the three parameters, namely, the zenith angle, $\Delta m^2/E$, and the mixing angle θ_3 of the electron neutrinos with tau neutrinos. The values of the triplet under the resonance is found numerically.

1 Introduction

It was almost ten years ago that the atmospheric neutrino anomaly[1] could be explained by the oscillations of muon neutrinos into tau neutrinos [2]. This interpretation has been confirmed by the observation of the zenith angle dependence of the neutrino fluxes by the SuperKamiokande[3]. While the zenith angle dependence of the electron-like events is consistent with the theoretical predictions[4][5], that of the muon-like events disagree with the theory, especially for the up-going events.

The neutrino state with flavor basis and that of mass basis is related to each other by the mixing matrix as follows,

$$\begin{pmatrix} \nu_e \\ \nu_\mu \\ \nu_\tau \end{pmatrix} = \begin{pmatrix} c_1c_3 & s_1c_3 & s_3 \\ -s_1c_2 - c_1s_2s_3 & c_1c_2 - s_1s_2s_3 & s_2c_3 \\ s_1s_2 - c_1c_2s_3 & -c_1s_2 - s_1c_2s_3 & c_2c_3 \end{pmatrix} \begin{pmatrix} \nu_1 \\ \nu_2 \\ \nu_3 \end{pmatrix}, \tag{1}$$

where $s_i = \sin\theta_i,\ c_i = \cos\theta_i \quad \text{for } i = 1, 2, \text{ and } 3$. The Super-Kamiokande shows that

$$P(\nu_\mu \to \nu_\mu) + P(\nu_\mu \to \nu_\tau) \simeq 1. \tag{2}$$

The equation (2) means that $s_3 \simeq 0$, which is consistent with the CHOOS results[6].

Once the the matter enhanced neutrino oscillations was proposed [7], it has soon been applied to the atmospheric neutrinos crossing the earth[8]. Recently much attention[9] has been devoted to this subject. However, it seems that no systmatic study has not been made yet, especially for the three flavor model of neutrinos.

The purpose of this paper is to reveal the conditions that amplify the effect of θ_3 through matter oscillations so that we can determine this small mixing angle.

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2 Matter Enhanced Atmospheric Neutrinos

I assume that the masses of neutrinos are hierarchical, namely, $m_1 < m_2 < m_3$, Furthermore I assume that the solar neutrino oscillations are attributed to $\delta m_{21}^2 \equiv m_2^2 - m_1^2$. The atmospheric neutrino oscillations are derived by $\delta m_{32}^2 \equiv m_3^2 - m_2^2$, the value of which is $\mathcal{O}(10^{-3}) eV^2$. The solar neutrino problem may be solved by the matter enhanced, or just so oscillations. In either case, δm_{21}^2 is much smaller than δm_{32}^2, and irrelevant to our concerns. In the following, we assign $\delta m_{21}^2 = 0$, and $\Delta m^2 \equiv \delta m_{31}^2 = \delta m_{32}^2$. The parameters contained in the model are θ_2, θ_3, and Δm^2.

In calculating the matter effect of the earth, I use the density profile given by the "Preliminary reference Earth model"(PREM)[10].

The prominent feature of our model is that the survival probability of electron neutrinos $P(\nu_e \to \nu_e)$ is completely determined by the three parameters, *i.e.*, θ_3, $\Delta m^2/E$ where E is the neutrino energy, and the zenith angle z. It is natural to define the resonance condition as

$$P(\nu_e \to \nu_e) = 0 \tag{3}$$

The resonance states form curves in the the three parameter space (θ_3, $\Delta m^2/E$, $\cos z$) corresponding to discretization of the resonance wavelength. I solve eq.(3) numerically, and show in Fig.1 and Fig.2 the values of the three parameters at the resonance. Fig.1 and Fig.2 correspond to the longest and the second longest wavelength, respectively.

The other probabilities $P(\nu_e \to \nu_\mu)$, $P(\nu_e \to \nu_\tau)$, $P(\nu_\mu \to \nu_\mu)$, and $P(\nu_\mu \to \nu_\tau)$ also depend on the $\nu_\mu \leftrightarrow \nu_\tau$ mixing angle θ_2.

3 Summary and Discussions

I have investigated the resonance conditions of neutrinos passing through the earth in a wide range of parameters, although only a small mixing angle $\theta_3 < 13°$ has a practical meaning as has been suggested by the CHOOS experiments.

The results show that it is necessary to observe the high energy electron neutrinos crossing the depth of the earth to investgate the small mixing angle θ_3. For example, the resonance occurs at $\theta_3 = 6.4°$, $\Delta m^2/E = 6.1 \times 10^{-4}\ eV^2/GeV$, and $\cos z = -0.9$. The energy of electron neutrino is 5 GeV if $\Delta m^2 = 3 \times 10^{-3} eV^2$. This indicates the need to assemble the data of the zenith angle distribution of the electron neutrinos whose energy is of order 5 GeV in order to determine the lepton mixing angle θ_3.

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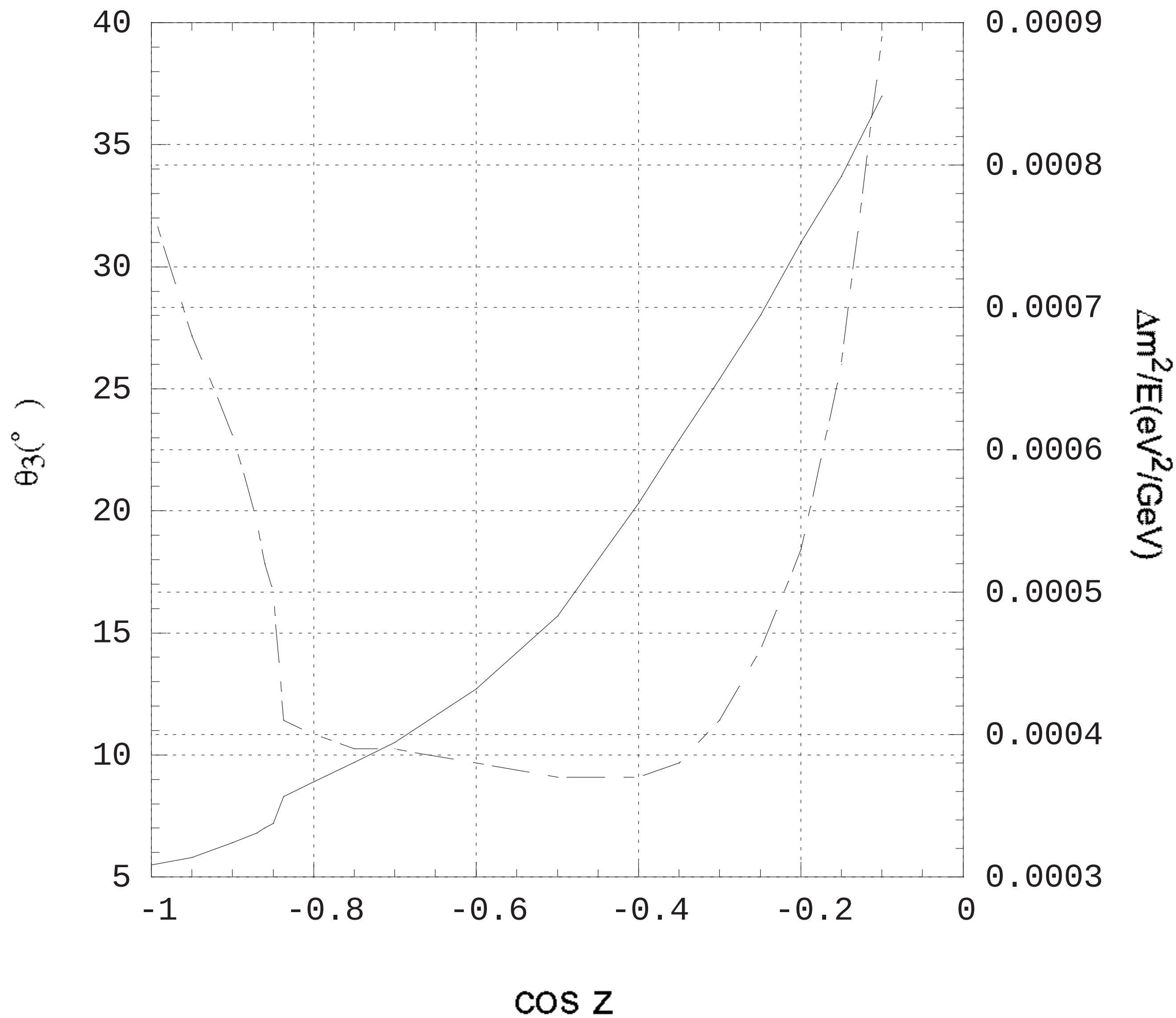
Figure Captions

Fig.1 The resonace conditions(I) as functions of $\cos z$. The solid line represents θ_3, and the dot-dashed line represents $\Delta m^2/E$.

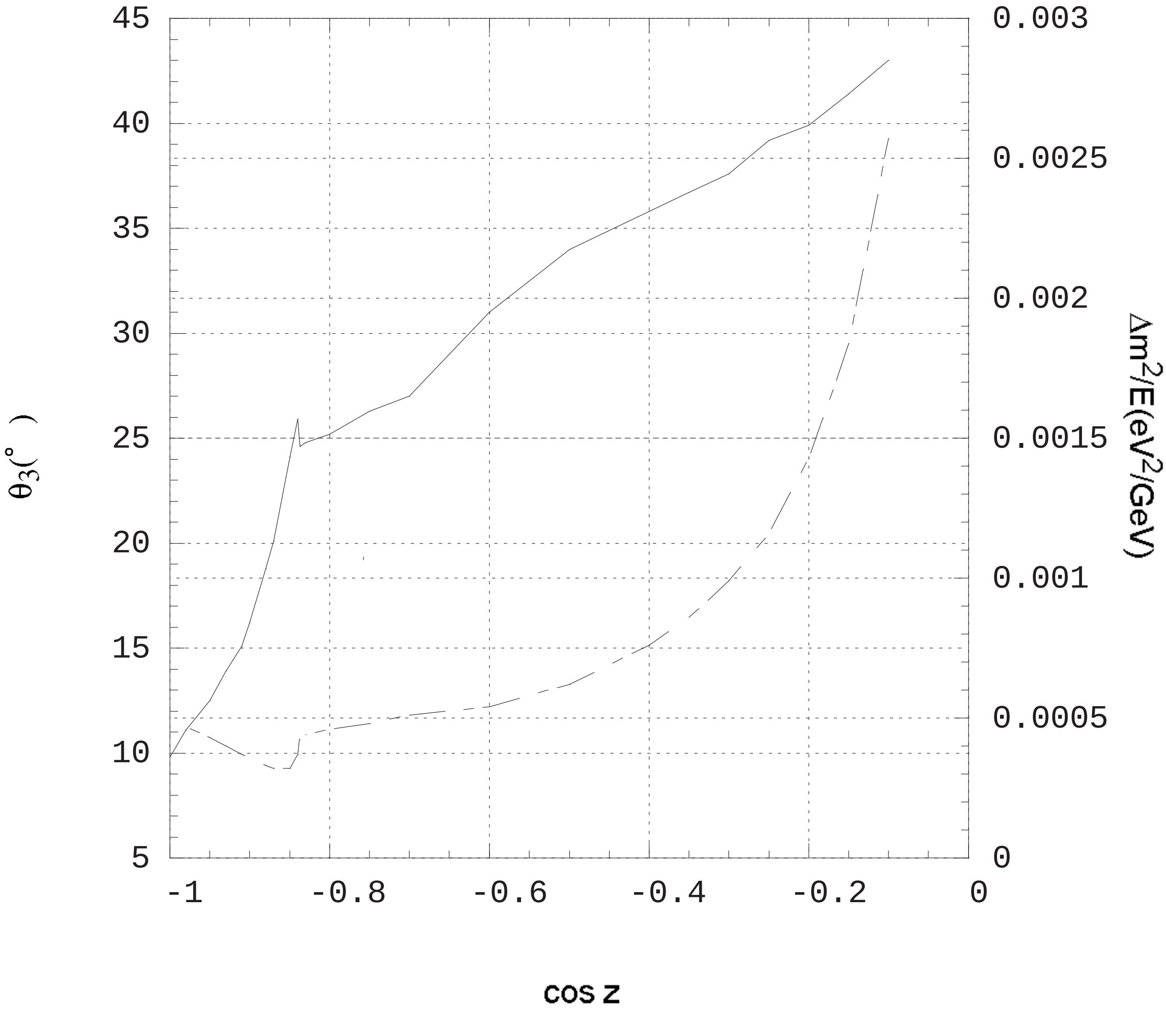
Fig.2 The resonace conditions(II) as functions of $\cos z$. The solid linerepresents θ_3, and the dot-dashed line represents $\Delta m^2/E$.

Fig.1



40
35
30
25
20
15
10
5
θ3(°)
0.0009
0.0008
0.0007
0.0006
0.0005
0.0004
0.0003
Δm2/E(eV2/GeV)
-1
-0.8
-0.6
-0.4
-0.2
0
cos Z

Fig.2



45
40
35
30
25
20
15
10
5
0.003
0.0025
0.002
0.0015
0.001
0.0005
0
-1
-0.8
-0.6
-0.4
-0.2
0
$\theta_3(°)$
$\Delta m^2/E(eV^2/GeV)$
cos z